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HEAT TRANSFER AND HYDRODYNAMICS NEAR CURVILINEAR SURFACES

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This paper surveys the state-of-the-art in the field of heat transfer and hydrodynamics near surfaces with longitudinal curvature.

1. INTRODUCTION

Flow and heat transfer on surfaces with longitudinal curvature are very common in various areas of engineering. The flows are induced as a result of special design features of technical facilities or are created specially to intensify heat and mass transfer or improve thermogasdynamic processes. Flows near curvilinear surfaces are most frequently encountered in aviation, rocket technology, heat engineering, power-plant engineering, shipbuilding, and internal combustion engines. The effects of longitudinal curvature in technical facilities are usually combined with the action of other factors, such as the longitudinal pressure gradient, injection, compressibility, external turbulence, etc.

Flows on surfaces with longitudinal curvature belong to the class of flows in fields of centrifugal body forces. Local and integral parameters of a curvilinear boundary layer undergo noticeable quantitative changes compared to a boundary layer on a flat plate even for $\delta^{**}/R_w > 0.001$, which is rather frequently encountered in technical facilities. In calculations curvature effects are taken into account both in the differential equations of motion and energy and in semiempirical relations for turbulent shear stresses and heat fluxes.

The outstanding features of curvilinear flows distinguishing them from a flow near a flat plate are:

1) the presence of a transverse pressure gradient;

2) active and conservative action of centrifugal forces, formation of secondary flows in the form of Görtler vortices;

3) specific features of transition from a laminar to a turbulent flow.

This necessitates the development of separate methods for calculating heat transfer and hydrodynamics that take rather full account of the above effects.

The problem of heat transfer and hydrodynamics near curvilinear surfaces has been under study for over sixty years. One of the first investigations was carried out by F. Wattendorf [1]; thereafter studies of various aspects of curvilinear flows were continued by H. Görtler [2], F. Kreith [3], and I. Tani [4]. In the last thirty years extensive theoretical and experimental advancements in this problem have been made by V. K. Shchukin [5], by B. P. Ustimenko [6], at the Institute of Technical Thermophysics of the National Academy of Sciences (ITTP NAS) of Ukraine [7], and at the Institute of Thermophysics of the Siberian Branch of the Russian Academy of Sciences (ITP SB RAS) [8]. Abroad, active investigations have been conducted by B. Mayle, R. So, C. Mellor, T. Symon, B. Moffat, and R. Goldstein (USA), P. Bradshaw and M. Gibson (England), and B. Shivaprasad and B. Pamaprial (India). A detailed list of publications in this field can be found in [7, 9, 10].

This survey contains an analysis of the current state of the problem of heat transfer and hydrodynamics near surfaces with longitudinal curvature (convex and concave). In view of space restrictions it includes only the most important scientific results obtained by now, accepted views on the state of the problem are considered, and prospects for further developments in it are given.

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2. STATE OF THE PROBLEM

2.1. Physical Similarity and Stability. The longitudinal curvature of a surface favors an increase in the curvature of streamlines and, as a consequence, leads to a change in the degree of fullness of the velocity and temperature profiles. Earlier investigations [7-9] showed that a decrease in the intensity of turbulent agitation decreases the degree of fullness of the velocity profile near a convex wall and increases the intensity of turbulent transfer near a concave wall (shear flow). For a semiinfinite jet the curvature of the surface exerts a similar effect in the shear portion of the velocity profile and influences only slightly the shape of the profile in its jet portion.

H. Görtler was the first to introduce a criterion of additional similarity between the processes of heat transfer and hydrodynamics near a curvilinear surface. Subsequently this dimensionless quantity was called the Görtler number:

$$\mathrm{G\ddot{o}} = \frac{u_\infty \delta}{\nu} \sqrt{\left(\frac{\delta}{R_w}\right)} = \frac{u_\infty R_w}{\nu} \left(\frac{\delta}{R_w}\right)^{1.5} . \tag{1}$$

The first factor on the right-hand side of Eq. (1) is the ordinary Reynolds number, and the second factor is the dimensionless curvature of the boundary layer. For a shear flow the additional condition of similarity (curvature parameter) has the form

$$\delta / R_w = \mathrm{idem} . \tag{2}$$

For a semiinfinite jet near a curvilinear wall this condition is written as

$$\delta_m / R_w = \mathrm{idem} , \tag{3}$$

where δ_m is the distance from the wall to the maximum of the longitudinal velocity component.

Thus, in the simplest case of a shear flow in the absence of external effects (turbulence, pressure gradient, etc.) the similarity equations for heat transfer and friction are represented as

$$\mathrm{Nu}/\mathrm{Nu}_0 = f_1\,(\delta / R_w) , \quad C_f / C_{f0} = f_2\,(\delta / R_w) . \tag{4}$$

For a semiinfinite jet one should use simplex (3) in the right-hand side of Eqs. (4).

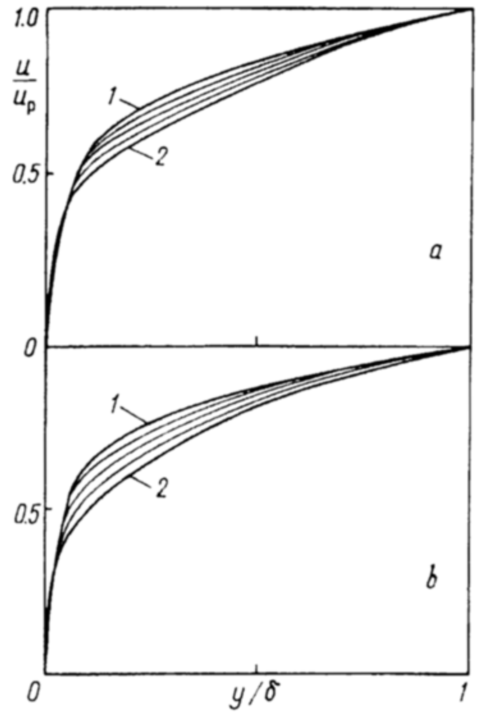
The character of the action of centrifugal forces on the structure of the flow depends on the mutual position of the vectors of the body force F and its gradient $\partial F / \partial n$. An active effect is observed in the case where these vectors are directed oppositely or are perpendicular. When the directions are the same, the action of centrifugal forces is conservative. In the first case turbulence is generated in the flow and secondary flows appear, whereas in the second case the opposite effect is observed. The secondary flows near curvilinear surfaces include Taylor–Görtler vortices that are formed in a boundary layer near a concave surface [2, 4, 5, 7], as well as a pair vortex in a curvilinear channel.

The Rayleigh method based on the concept of an ideal flow makes it possible to obtain two criteria of stability [5]:

$$\frac{d}{dr}\,[\rho\,(ur)^2] > 0 , \quad \frac{d}{dr}\,[\rho u / r] > 0 . \tag{5}$$

The first of these is based on the hypothesis of conservation of velocity circulation $\Gamma = ur$ in random radial movement of small volumes, and the second is based on the hypothesis of conservation of the angular velocity of rotation $\omega = u/r$.

L. Prandtl in 1929 was the first to point out the formal analogy between the processes of heat and momentum transfer in a curvilinear flow and in a temperature-stratified medium. Proceeding from this analogy, P. Bradshaw obtained a centrifugal analog of the Richardson number:



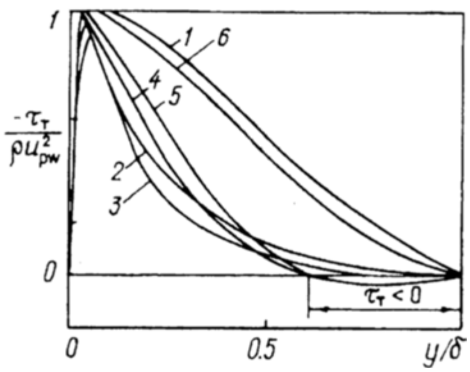


Fig. 1. Adaptation of the profile of the longitudinal velocity in transition from a plane to a convex surface (a, $\delta_0 = 0$; b, $\delta_0 \neq 0$): a) 1, $\delta^{**}/R_w = 0.001$; 2, $\delta^{**}/R_w = 0.0054$; b) 1, plane surface; 2, $\delta^{**}/R_w = 0.006$.

Fig. 2. Adaptation of turbulent shear stresses in transition from a plane to a convex surface ($\delta_0^{**}/R_w = 0.096$): 1) plane surface; δ^{**}/R_w: 2) 0.01, 3) 0.0109, 4) 0.0119, 5) 0.0133; 6) $\delta_0^{**} = 0$.

$$\mathrm{Ri} = 2\frac{\Gamma}{r}\Big/\frac{\partial \Gamma}{\partial r}, \tag{6}$$

which is the ratio of turbulence generation by centrifugal forces to turbulence generation by the transverse gradient of the velocity in a shear flow. A flow in a field of centrifugal forces a stable when $\mathrm{Ri} > 0$ and unstable when $\mathrm{Ri} < 0$. Other expressions for the Richardson number are also used:

$$\mathrm{Ri} = 2S\,, \quad \mathrm{Ri} = 2S\,(1 + S)\,,$$

where $S = (u/R_w)/(\partial u/\partial y)$. Under nonisothermal conditions the Richardson number has the form [8]

$$\mathrm{Ri} = \mathrm{Ri}_0 \left[1 + \frac{(\psi - 1)}{2\,(1 + \psi n^{-1})}\right], \tag{7}$$

where $\psi = T_w/T_f$ is the temperature factor; n^{-1} is the exponent in the power-law profile of the velocity; Ri_0 is the Richardson number for isothermal conditions.

For a compressible flow the Richardson number is written as [10]

$$\mathrm{Ri} = \mathrm{Ri}_0 \left[1 + \frac{k - 1}{2}\,\mathrm{M}_\infty^2 + 0.25\,\mathrm{M}_\infty^2\,\mathrm{Ri}_0^2\right],$$

where M_∞ is the Mach number of the external flow, and k is the specific-heat ratio.

2.2. Adaptation and Relaxation of a Boundary Layer. *Adaptation* is the transformation of a flow in transition from a plane to a curvilinear surface, whereas *relaxation* is the transformation in transition of a flow from a curvilinear to a plane surface. In both cases there is a gradual change in the turbulent, local, and integral parameters of the flow from the laws governing a plane/curvilinear flow to the laws governing a curvilinear/plane flow that terminates at a certain length of the surface.

2.2.1. Adaptation of a Boundary Layer. In transition from a plane ($1/R_w = 0$) to a curvilinear ($1/R_w \neq 0$) surface three main effects appear:

1) The dR_w/dx-effect associated with an instantaneous change in the surface curvature radius.

2) The effect of a local longitudinal pressure gradient appearing due to a change in the transverse pressure gradient from a zero value near a plane wall to a nonzero value on a curvilinear surface.

3) The effect of the wall curvature proper (the R_w-effect).

The first two factors have a noticeable effect over small distances from the inlet and the third factor dominates over large distances. The length of the curvilinear surface at which the action of the first two factors terminates is called the *adaptation length* of the boundary layer. Beyond this length there is the *main portion* of the flow.

Investigations devoted to the physical foundations of boundary layer adaptation on a curvilinear surface are very few in number. At the present time only a few works dealing with a convex wall have been published [11, 12], whose results are considered below.

Adaptation of a turbulent boundary layer in transition from a plane to a convex surface occurs differently for zero ($\delta_0 = 0$) and nonzero ($\delta_0 \neq 0$) boundary layer thicknesses at the inlet (Fig. 1). In the first case the fullness of the velocity profile near the surface increases insignificantly and that in the external portion of the profile decreases; in the second case the fullness of the velocity profile over the convex-wall length decreases in conformity with the convex-curvature effect. Adaptation of the velocity profile terminates at the length corresponding to the condition [11]

$$\delta_a^{**}/R_w = 0.0035 + 1.69\,(\delta_0^{**}/R_w)^{0.92}\,, \tag{8}$$

where δ_a^{**} and δ_0^{**} are the momentum loss thickness at the end of the adaptation length and at the beginning of the convex wall, respectively.

Investigations of turbulent characteristics over the adaptation length show that the distribution of turbulent friction over the boundary layer cross section at $\delta_0 = 0$ and $\delta_0 \neq 0$ differ substantially. When $\delta_0 = 0$, turbulent friction decreases equidistantly over the entire cross section of the boundary layer (lines 1 and 6, Fig. 2) in conformity with the convex-curvature effect. When $\delta_0 \neq 0$, after the transition of the flow from a plane to a convex surface, turbulent friction τ_t first decreases sharply and then gradually and asymptotically approaches the form characteristic for $\delta_0 = 0$. The rate of decrease of τ_t increases with increase in the curvature factor δ_0^{**}/R_w, and at a rather large value of the latter parameter a region of negative friction appears in the outer part of the boundary layer (Fig. 2). This is explained by the fact that at large values of δ/R_w the term of turbulence generation in the balance equation for shear stresses

$$G_\tau = \overline{v'^2}\,\frac{\partial u}{\partial y} - \left(2\overline{u'^2} - \overline{v'^2}\right)\frac{u}{R_w}$$

acquires a negative value. Analysis shows that the appearance of negative values $-\overline{u'v'}$ in the outer part of the boundary layer is caused not by the effect of significant curvature (as interpreted by many research workers), but rather by transition from a plane to a convex surface at large values of δ/R_w and the associated transformation of the turbulent structure of the flow. From the physical point of view the appearance of a zone of negative turbulent friction testifies to transition of energy of fluctuational motion to energy of averaged flow ("negative" turbulent viscosity). This also occurs in other types of flows in a field of centrifugal forces [6].

The fact of the appearance of a zone with negative turbulent shear stresses was discovered in [13]. That work presents detailed experimental data on turbulent characteristics of heat and momentum transfer over the adaptation length near a convex wall. Measurements showed that immediately after the transition to the convex wall the turbulent Prandtl number is equal to 1.15–1.2 near its surface and decreases to 0.5–0.6 in the external region.

The coefficients of friction and heat transfer over the adaptation length near a convex surface of constant curvature at $\delta^{**}/R_w = 0-0.013$ are calculated from the equations

$$(C_f/C_{f0})_{Re^{**}} = [1 + 10^3 (\delta^{**} - \delta_0^{**})/R_w]^{-0.31}, \tag{9}$$

$$(St/St_0)_{Re_T^{**}} = [1 + 10^3 (\delta_T^{**} - \delta_{T0}^{**})/R_w]^{-0.33}. \tag{10}$$

Some results of investigation of heat transfer over the adaptation length near a convex surface are also given in [13]. In particular, it is shown there that in the adaptation region $St \sim Re_T^{**-1}$, which virtually corresponds to a laminar regime.

In calculations of a turbulent boundary layer over the adaptation length the changes in the flow curvature in transition from a plane to a convex surface are taken into account by using the effective curvature radius R_{ef}. The equations suggested at the ITTP NAS of Ukraine [11] and the ITP SB RAS [8] have the following form:

$$R_{ef}^{-1} = R_w^{-1} [1 - \exp(-x/R_w)], \tag{11}$$

$$R_{ef}^{-1} = R_w^{-1} [1 - \exp(-0.1x/R_w)]. \tag{12}$$

2.2.2. Boundary Layer Relaxation. Problems of heat transfer and hydrodynamics over the relaxation length beyond a convex wall are considered in detail in [12, 13]. After transition to a plane surface the flow recovers much more slowly than it undergoes adaptation under the influence of curvature. Faster recovery of turbulent friction occurs in the outer part of the profile than near the surface. A turbulent heat flux behaves differently; the transverse correlation $\overline{v'T'}$ along the relaxation length gradually increases compared to the convex wall and even exceeds its value for a plane surface. This is followed by an asymptotic reverse approximation to the laws of a plane wall. The turbulent Prandtl number over the relaxation length is characterized by considerable scatter of experimental data; the value of Pr_T is still somewhat lower at a considerable distance from a convex wall than for a plane wall and amounts to 0.5–0.6 in the region with $y/\delta = 0.4-1.0$. The mixing path length at the beginning of the relaxation length is 2–3 times smaller than on a plane surface [14].

2.3. Shear Flow. Main Section. Nongradient Flow. The overwhelming majority of investigations into the structure of a curvilinear turbulent boundary layer over the main portion of a flow indicate that due to a decrease in turbulent exchange the boundary layer thickness on a convex surface is smaller than on a plane surface under otherwise equal conditions. The reverse is the case on a concave surface. The conventional boundary layer thicknesses δ^*, δ^{**}, and δ_T^{**} change correspondingly (they are smaller on a concave wall and larger on a convex wall compared to a plane one [7]).

In the case of a power-law approximation of the velocity profile $u/u_\infty = (y/\delta)^{1/n}$ the value of the exponent n is determined from the equations [7, 8]

$$R_w > 0, \quad n/n_0 = 1 - 4.34 (\delta^{**}/R_w)^{0.45}, \tag{13}$$

$$R_w < 0, \quad n/n_0 = 1 + 2.69 (\delta^{**}/|R_w|)^{0.65}. \tag{14}$$

Then, from these equations one can obtain expressions for the relative form parameter $\overline{H} = H/H_0$.

In a boundary layer near a curvilinear surface a universal logarithmic law is maintained for the velocity and temperature profiles in a certain region near the wall [9]. The width of this region is reduced on a convex surface compared to a plane wall and is enlarged on a concave surface. Convex curvature favors an increase in the relative thickness of the viscous sublayer ξ_1 and a decrease in the velocity ω_1 and temperature θ_1 on its boundary.

Universal velocity and temperature profiles in the region of the logarithmic law are located higher for a convex wall and lower for a concave wall than for a plane surface. Equations for calculating velocity and temperature profiles in the logarithmic region as well as over the entire cross section with the aid of the Coles profile are given in [9].

Longitudinal curvature exerts a strong effect on the turbulent structure of a flow. Convex curvature decreases normal stresses, while concave curvature increases them, and the degree of this change depends on the ratio δ/R_w [9]. The correlation $\overline{u'v'}$ is decreased on a convex surface and increased on a concave one compared to a plane surface. In the case of moderately convex curvature the distribution of shear stressess over the boundary layer cross section is described satisfactorily by the equation [7]

$$\bar{\tau} = \bar{\tau}_0 \frac{\Phi\xi \exp(\Phi\xi)}{\mathrm{sh}(\Phi\xi)} \left[1 + U \frac{(\Phi\xi)^2}{1 + \Phi\xi}\right], \tag{15}$$

where $U = 0.8 \left[1 + \exp\left(-\frac{E}{1 + 0.2E}\right)\right] \exp(2\xi\,|\Phi|)$; $\Phi = \Lambda - 2\delta/R_w$; $E = (|\Phi|\,\xi)^{0.5} \exp(0.2\xi\,|\Phi|)$; $\Lambda = \frac{\delta}{\tau_w}\frac{dP}{dx}$ is the parameter of the pressure gradient.

Turbulent viscosity, just like other turbulent characteristics, is increased on a concave surface and decreased on a convex one. The influence of curvature on the thermal turbulent characteristics is similar to its effect on the hydrodynamic turbulent parameters.

The curvature of the surface exerts a nonlinear effect on surface friction and heat transfer. In the case of moderately convex curvature ($\delta/R_w = 0.01$) the value of c_f is 10–20% smaller than on a plane surface, and at $\delta/R_w = 0.1$ this decrease is already equal to 25–35%. Concave curvature exerts an opposite effect on surface friction. For a convex surface it was obtained that $c_f \sim \mathrm{Re}^{**-1}$ at $\delta/R_w \approx 0.1$ and $c_f \sim \mathrm{Re}^{**-0.48}$ at $\delta/R_w = 0.01-0.015$; this is an intermediate case between a boundary layer on a plane wall ($c_f \sim \mathrm{Re}^{**-0.25}$) and significant curvature.

In a thermal boundary layer $\mathrm{St} \sim \mathrm{Re}_T^{**-0.55}$ for moderate curvature ($\delta/R_w = 0.01$) and $\mathrm{St} \sim \mathrm{Re}_T^{**-1}$ for significant curvature, indicating approximately the same effect of curvature on friction and heat transfer.

To describe the effect of curvature on surface friction and heat transfer various equations that can be found in [7] have been suggested. For a convex surface under quasi-isothermal conditions the formulas suggested at the ITTP NAS of Ukraine correspond most fully to the nonlinear character of the curvature effect:

$$\Psi_R = \begin{cases} (1 + 2900\delta^{**}/R_w)^{-0.08}, & \delta^{**}/R_w = 0 \ldots 0.003, \\ 0.86 - 10^4\,\delta^{**}/R_w, & \delta^{**}/R_w = 0.003 \ldots 0.008; \end{cases} \tag{16}$$

$$\Psi_R = (1 + 10^3\,\delta^{**}/R_w)^{-0.12}; \quad \delta^{**}/R_w = 0 \ldots 0.011. \tag{17}$$

For a concave surface under nonisothermal conditions the most reliable equation for Ψ_R has the form [8]

$$\Psi_R = \left[1 + 1800 \frac{\delta^{**}}{R_w}\left(1 + \frac{\psi - 1}{2(1 + \psi n^{-1})}\right)\right], \tag{18}$$

where $\psi = T_w/T_f$ is the temperature factor; n is the exponent in the equation for the velocity profile $\omega = \xi^n$. Here the combination in round brackets takes account of an additional effect caused by centrifugal temperature stratification.

For the relative function of heat transfer Ψ_R^T the Reynolds analogy can be used in the form $\Psi_R = \Psi_R^T$ [7]. With allowance for the difference of the Prandtl number from unity, the following equations were obtained in [8]:

$$\Psi_R^T = \left\{1 + a\,\mathrm{Pr}^{-1.2}\,\delta_T^{**}/R_w \left[1 + \frac{\psi - 1}{2(1 + \psi n^{-1})}\right]\right\}^b, \tag{19}$$

where $a = 2200$ and $b = -0.115$ for a convex wall and $a = 1800$ and $b = 0.162$ for a concave one. A comparison of Eqs. (16)-(19) with experimental data obtained by M. Gibson, R. So, B. Mayle, P. Bradshaw, T. Symon, B. Moffat, and other authors is given in [7, 9].

2.4. Shear Flow. Main Section. Effect of Various Factors. Under actual conditions, flow and heat transfer near curvilinear surfaces are compounded by various factors such as pressure gradient, compressibility, nonisothermicity, injection through a porous wall, etc.

2.4.1. Longitudinal Pressure Gradient. The effect of positive and negative longitudinal pressure gradients on local, turbulent, and integral characteristics of a boundary layer near convex and concave surfaces are considered in detail in [7, 9]. Of the main results obtained we can mention here that:

1) on a convex surface, when $dP/dx > 0$, the critical parameter of flow separation is decreased considerably due to a decrease in the boundary layer thickness;

2) in the region of the joint effect of flow acceleration and convex curvature $\mathrm{St} \sim \mathrm{Re}_T^{**-2}$, indicating the additivity of the effect of these factors. This is due to the fact that convex curvature smoothes out turbulence in the outer region of the boundary layer, while acceleration causes growth of the viscous-sublayer thickness;

3) in calculations of friction and heat transfer the additivity principle is applicable if a correction is used that characterizes the effect of curvature on the critical parameter of the pressure gradient [7].

2.4.2. Compressibility. The effects of the compressibility of a flow on convex and concave surfaces under conditions of a pressure gradient were studied in [15, 16]. For air with $M_\infty < 4.0$ the relative functions of curvature for convex and concave walls have the form

$$R_w > 0, \quad \Psi_R = (1 + 2200\,\delta^{**}/R_w)^{-0.16}, \quad \Psi_R^T = (1 + 2200\,\delta^{**}/R_w)^{-0.14};$$

$$R_w < 0, \quad \Psi_R = (1 + 1800\,\delta^{**}/R_w)^{0.19}, \quad \Psi_R^T = (1 + 1800\,\delta^{**}/R_w)^{0.18}$$

and with an error not exceeding 20% they agree with the equations for Ψ_R and Ψ_R^T for an incompressible flow. Thus, the curvature effects are approximately identical for subsonic and supersonic regimes of flow.

When $dP/dx > 0$, the boundary layer separation for convex and concave surfaces is determined by the equations

$$\Lambda_{0\,\mathrm{cr}}^R/\Lambda_{0\,\mathrm{cr}} = (1 + 1150\,\delta^{**}/R_w)^{-1.1}, \quad \Lambda_{0\,\mathrm{cr}}^R/\Lambda_{0\,\mathrm{cr}} = (1 + 1150\,\delta^{**}/R_w)^{0.23}.$$

The equation corresponding to a convex wall differs substantially from the results obtained for an incompressible flow, indicating a stronger effect of surface curvature on boundary layer stability at large Mach numbers.

Equations for the relative functions of friction and heat transfer on a convex/concave surface when $dP/dx \le 0$ are presented in [15, 16]. Analysis of these equations shows that the principle of the additivity of curvature and pressure gradient can also be used for a compressible flow with account taken of the correction to the critical parameter of flow separation for the curvature. This result seems to be extremely important, since it allows one to substantially simplify the calculation procedure and use the algorithms of a plane turbulent flow.

2.4.3. External Turbulence. The influence of external turbulence on heat transfer and hydrodynamics near a curvilinear surface was investigated only in [7, 13]. The main results of these investigations are:

1) just as on a plane surface, the external turbulence near a convex wall increases the fullness of the velocity profile and decreases the viscous-sublayer thickness;

2) the shear stresses are increased over the entire thickness of the boundary layer; in the outer part negative values of τ_t were discovered even at $\delta/R_w = 0.03$ due to the negative value of the generation term G_τ in the balance equation for shear stresses;

3) the turbulent characteristics $\overline{T'^2}$, $\overline{v'T'}$, and $\overline{u'T'}$ in the presence and absence of external turbulence near a convex surface virtually coincide, indicating the dominance of the curvature effect over the effect of external turbulence;

4) in the region with $\delta/R_w \le 0.08$ the values of Ψ_{Tu} are correlated satisfactorily by the equations

$$\Psi_{Tu} = 1 + 0.021\ Tu^{1.75} \quad \text{at} \quad Tu = 0 \ldots 3\ \%\ ,$$

$$\Psi_{Tu} = 0.922\ Tu^{0.13} \quad \text{at} \quad Tu = 3 \ldots 7\ \%\ .$$

They virtually coincide with results obtained on a flat plate, i.e., in the present case too one can apply the principle of additivity of curvature and external turbulence.

2.4.4. Injection through a Porous Wall. There are virtually no works devoted to the effect of porous injection through a curvilinear wall on friction and heat transfer. Individual results of a theoretical calculation on this problem are presented in [17]. They show that:

1) convex curvature decreases the critical parameter of injection. The value of this parameter on a convex surface is determined by the equation $b_{cr} = b^0_{cr}[1 + 10^3\delta^{**}/R_w]^{-0.52}$. This equation can also be used for determining the thermal critical parameter of injection;

2) when using the above formula, it is possible to employ relative functions of injection obtained for a plane surface. In this case the condition of additivity of injection and curvature is satisfied;

3) injection exerts a major influence on viscous and thermal sublayers; the action of curvature is concentrated mainly in the outer part of the boundary layer.

2.4.5. Nonisothermicity. Centrifugal forces in a boundary layer favor strengthening of turbulent transfer near a concave wall and its weakening near a convex wall. The corresponding effect in the case of differences between the temperatures of a flow and a wall is taken into account by the term in square brackets in Eq. (19). It is recommended that the "pure" effect of nonisothermicity be determined from relations for a plane wall [8].

2.5. Semiinfinite Jet and Gas Screen. The hydrodynamics of a semiinfinite jet near a curvilinear surface has specific features. In this case near the curvilinear surface one observes regions with an active and conservative character of the action of centrifugal forces. Near a convex wall the zone of active effect is located in the shear portion of the profile, while outside it there is a zone of conservative effect. The reverse is the case for a concave wall. As a result, there is a transport of energy of turbulence from the region of active to the region of conservative effect, which is reflected in the distribution of turbulence, flow, and heat transfer.

A few investigations of the heat transfer and hydrodynamics of jet flows near curvilinear surfaces were mainly carried out at the ITTP NAS of Ukraine [18, 19], as well as at the University of Minnesota under the leadership of Prof. R. Goldstein. Due to the action of centrifugal forces the fullness of the velocity profile in its shear portion increases near a concave surface and decreases near a convex one. The equation for calculating the exponent n in this region has the following form ($\delta_m/R_w < 0.020$) [18, 19]:

$$n/n_0 = 1 + a\,(\delta_m/R_w)^b\ , \tag{20}$$

where $a = -1.7$, $b = 0.31$ for a convex wall; $a = 2.69$, $b = 0.85$ for a concave wall ($n_0 = 12$). In the jet portion of the profile the curvature hardly changes the distribution velocity and corresponds to a plane semiinfinite jet.

The temperature profile near a concave surface is described satisfactorily by the power-law relation $\theta = (y/\delta_T)^{1/m}$, where $m = m_0[1 + 4.4(\delta_m/|R_m|)^{0.56}]$. Some additional data characterizing the hydrodynamics of flow near convex and concave surfaces can be found in [18, 19].

For calculating local heat transfer near convex and concave walls the following equation is used [18, 19]:

$$(Nu_x/Nu_0)_{Re_x} = 1 + a\,[\delta_m/R_w - b]^c\ , \tag{21}$$

where $a = 4.149$, $b = 0.002$, $c = 0.65$ for a concave wall; $a = -23.69$, $b = 0.0005$, $c = 0.81$ for a convex wall.

The longitudinal curvature of a surface influences substantially the *efficiency of a gas screen.* Convex curvature stabilizes a flow and thus increases the efficiency of a gas screen compared to a plane surface; concave curvature exerts the opposite effect.

The theoretical and experimental investigations of the efficiency of a gas screen near curvilinear surfaces are few in number. Individual aspects of this problem were investigated by B. Mayle, A. V. Shchukin, and M. Blair, at the ITTP NAS of Ukraine [7], and at the ITP SB RAS [8]. In [7] it was substantiated experimentally that the efficiency of a gas screen in the case of slit injection in front of a convex surface is determined by the equation for a plane surface with allowance for the curvature factor:

$$\eta = \left[1 + 0.24 \frac{x}{ms} \mathrm{Re}_s^{-0.25} \Psi_R\right]^{-0.8}, \tag{22}$$

where m is the injection parameter; the quantity $\Psi_R = (c_f/c_{f0})_{\mathrm{Re}^{**}}$ is determined from Eqs. (16) and (17). Allowance for the influence of flow acceleration and external turbulence on the efficiency of a gas screen is made by using corrections given in [7]. It is also shown in that work that the difference in the efficiency of a gas screen for discrete injection and continuous injection through a slit corresponds to the laws of a flat plate.

There are no reliable equations in the literature for calculating the efficiency of a gas screen near a concave surface. Taking into consideration results obtained by A. V. Shchukin (Kazan' Aviation Institute), as well as results obtained at the ITTP NAS of Ukraine, it is possible to recommend using Eq. (22) with allowance for Eqs. (18) and (19).

2.6. Semiempirical Hypotheses. Various models are used to calculate turbulent shear stresses and heat fluxes over the cross section of a boundary layer. According to the mixing path theory for curvilinear flows, the value of τ_t is determined from the equation

$$\tau_t = \rho l^2 \left(\frac{\partial u}{\partial r} \pm \frac{u}{r}\right)^2. \tag{23}$$

The plus sign corresponds to the hypothesis of conservation of velocity circulation, and the minus sign corresponds to the hypothesis of conservation of the angular velocity of rotation. In this case the mixing path length l is a function of the curvature of the streamlines.

P. Bradshaw suggested the following equation for determining the relative mixing path length near a curvilinear surface:

$$l/l_0 = (1 \pm \beta\,\mathrm{Ri})^{\mp N}, \tag{24}$$

where N is a constant. Its drawback is an instantaneous change in l with a jumpwise change in the surface curvature. Allowance for this factor in theoretical models is made by using an effective curvature radius determined by formulas (11) and (12).

Basic equations suggested for calculating the ratio l/l_0 are given in [7]. In a number of cases the difference between them attains 200–300%. For a shear flow near a convex surface relations that agree satisfactorily with the majority of the data published [7] are given below:

$$N = 1, \quad \beta = 10\,[1 + 8 \cdot 10^3 (\delta^{**}/R_w - 10^{-3})]^{-0.46};$$

$$N = -1, \quad \begin{cases} \beta = 2 - 2.96\,\mathrm{th}\,[0.72\,(10^3 \delta^{**}/R_w - 1)], \\ \beta = 10\,(1 + 1230\,\delta^{**}/R_w)^{-0.71}. \end{cases}$$

The first of the equations for $N = -1$ is satisfied in the region of $\delta^{**}/R_w = 0.001$–0.003 and the second at $\delta^{**}/R_w = 0.003$–0.008.

For conditions of significant curvature near a convex surface with a zone of negative friction in the outer part of the boundary layer A. A. Khalatov, E. É. Ikonnikova, and A. V. Kuz'min suggested the following formula [11, 17]:

$$l/l_0 = \exp(-30\,\mathrm{Ri}) - 3.5\,\mathrm{Ri}^2 + 1.2\,\mathrm{Ri} + 0.3. \tag{25}$$

In [8], on the basis of a theoretical approach, equations are suggested for convex and concave surfaces that also agree well with experimental data:

$$R_w > 0\,,\quad l/l_0 = \left[1 + \left(\frac{y}{l_0}\right)^2 \mathrm{Ri}\right]^{-0.25}; \tag{26}$$

$$R_w < 0\,,\quad l/l_0 = \left[1 - \left(\frac{y}{l_0}\right)^2 \mathrm{Ri}\right]^{0.25}. \tag{27}$$

V. Sovershennyi [7] suggested taking account of curvature effects in the formula for the linear dependence of the mixing path length near a wall [7] ($l = ky$):

$$R_w > 0\,,\ \ k/k_0 = 1 - 4.925\,\delta/R_w\,;\ \ R_w < 0\,,\ \ k/k_0 = 1 - 23.075\,\delta/R_w\,.$$

However, such an approach seems to be insufficiently accurate for calculating processes of heat transfer and hydrodynamics.

In [16] an attempt was made to take account of the effect of curvature on turbulent viscosity in a compressible flow by using the equation $\mu_t = \mu_t^0 \varepsilon_R$, where $\varepsilon_R = 1 - \beta\mathrm{Ri}$, with the Ri number being defined by an expression given in Sec. 2.1. The parameter β depends on the specific-heat ratio k and the Mach number M_∞ [15, 16]. The effect of curvature on the turbulent Prandtl number was investigated by R. So [7]. It was shown that it was insignificant for the main section of the flow. In especially accurate calculations allowance for the effect of curvature on the Prandtl number $\mathrm{Pr_T}$ can be made by using an equation suggested in [7].

In recent years various authors have extended the $k-\varepsilon$-model of turbulence to the case of curvilinear flows. Allowance for the effect of the curvature of the streamlines on turbulent transfer of momentum is made in the differential equation for ε by means of a correction introduced into the generation term using the equation $C_1 = 1.44 + \mathrm{Ri}$. The coefficient C_1 is increased in the region of suppression of turbulence ($\mathrm{Ri} > 0$) and decreased in the zone of its intensification ($\mathrm{Ri} < 0$).

In semiinfinite jets, in the region of shear flow before the point where $\tau_t = 0$ turbulent shear stresses are determined by Eq. (23), where $l = l_0(1 + a\mathrm{Ri})^b$. For a convex wall $a = 8.3$ and $b = -1$; for a concave wall $a = -3.85$ and $b = 0.75$ [18, 19]. In the jet portion of the profile the determination of turbulent friction is based on the hypothesis of constancy of the mixing path length $\tau_t = \rho\kappa(b - \delta_m)(u_m - u_{0.5})\partial u/\partial y$. Here b is the halfwidth of the jet [7], and $u_{0.5}$ is the velocity at the point $y = b$. For a curvilinear boundary layer the turbulence constant $\kappa = \kappa_0[1 + c(\delta_m/R_w)^g]$; in this case $c = 13.8$ and $g = 0.8$ near a convex wall and $c = -9.75$ and $g = 0.78$ near a concave wall [18, 19]. Certain additional data that characterize normal turbulent stresses, the coordinate of the point of zero turbulent friction, specific features of diffusion of turbulence from the region of active to the region of conservative effect of centrifugal forces, and other data are presented in [18, 19].

2.7. Centrifugal Instability. One of the specific features of the hydrodynamics near curvilinear surfaces is the appearance of Görtler longitudinal vortices in the boundary layer near a concave surface due to disturbance of the balance between the centrifugal force and the transverse pressure gradient. Such vortices were first discovered theoretically by H. Görtler [2] in a laminar regime of flow and by I. Tani in a turbulent flow [4]. Although a long period of time has passed since their discovery, some problems of heat transfer, surface friction, and formation and development of Görtler vortices have been investigated insufficiently as yet. The basic difficulty is accurate calculation of the coefficients of heat transfer and friction across a concave wall due to their wavy variation.

In the last fifteen–twenty years various aspects of heat transfer and hydrodynamics associated with Görtler vortices have been investigated in detail in the USA, Japan, and Germany. In Ukraine similar investigations have been carried out at the Institutes of Hydrodynamics and Technical Thermophysics of the National Academy of Sciences. In recent years, at the ITTP NAS of Ukraine linear and nonlinear approaches to analysis of centrifugal instability have been worked out that are based on development of the Taylor method [20].

In [20], in the case of a linear approximation with allowance for all the terms in the equation of motion, rather accurate relations were obtained for the critical Görtler number that determines the appearance of vortices in a laminar flow:

$$\mathrm{G\ddot{o}}_{cr} = 22.97 \quad \text{at} \quad \delta/R_w = 0 \ldots 0.02\,,$$

$$\mathrm{G\ddot{o}}_{cr} = 22.76\,(\delta/R_w)^{0.05} \quad \text{at} \quad \delta/R_w = 0.02 \ldots 0.1\,.$$

A comparison of these equations with earlier published theoretical calculations and experiments of Görtler, Meksin, Aihara, Kahavita, Smith, et al. that was carried out in [21] shows that the approach developed in [20] provides a good description of the majority of the experimental data (Görtler obtained the value $\mathrm{G\ddot{o}}_{cr} = 16$) and takes into account the slight change in the critical Görtler number as a function of the curvature δ/R_w. Satisfactory agreement between the linear theory and experiments was also obtained for the wave number (the distance between the axes of the vortices).

The linear approach makes it possible to predict on the whole a harmonic change in the coefficients of friction and heat transfer across a concave surface. However, with such an approach, on the average a value of the surface friction equal to the friction under conditions of nonvortical flow is maintained. This drawback is eliminated by allowance for the quadratic terms in the disturbing amplitudes. This nonlinear approach was developed in [20]. Use of the nonlinear approach makes it possible to obtain more accurate data on heat transfer and surface friction. Calculations carried out in [20] show that the effect of vortices on surface friction and heat transfer in a laminar flow is very appreciable and may attain 80–90%. This effect depends little on the surface curvature δ/R_w.

Investigations devoted to Görtler vortices in a turbulent flow are very scarce. Suffice it to say that up to now there are no reliable data on the value of the critical Görtler number corresponding to the appearance of vortices in a turbulent boundary layer. In [21] an equation for the critical Görtler number is given that is obtained with allowance for the effect of the curvature of the streamlines on the turbulent viscosity. In the region with δ/R_w = 0.01–0.02 it has the form

$$\mathrm{G\ddot{o}}_{cr} = 3843 - 68520\,\delta/R_w\,.$$

The value of $\mathrm{G\ddot{o}}_{cr}$ decreases with an increase in δ/R_w, just like the wave number [21].

Results of investigation of the effect of a longitudinal pressure gradient, injection and suction from the surface, and external turbulence of the flow on heat transfer, surface friction, the wave number, and the profiles of the disturbing amplitudes are presented in works that were carried out in recent years at the ITTP NAS of Ukraine [22-26].

2.8. Calculation Methods. Heat transfer and hydrodynamics near curvilinear surfaces are calculated by methods based on the solution of differential and integral equations of motion and energy. In the differential equations, the curvature of the streamlines leads to the appearance of the factor $(1 + y/R_w)$, which characterizes the curvature radius of the streamlines, the additional terms $2\tau/\rho R_w$ and $q/\rho c_p R_w$ in the equation of motion for the longitudinal coordinate and the energy equation, and an additional relation that characterizes the transverse pressure gradient $\partial p/\partial y$. Moreover, the curvature effects are reflected in the expression for τ, as well as in the semiempirical relations for turbulent friction and heat flux [7, 9, 10], In the integral equation of motion terms appear that characterize the pulsational components across the boundary layer and the boundary layer curvature; the integral expressions for the thermal boundary layer for curvilinear and plane flows agree with each other [9, 10].

Among the popular and rather simple methods of calculation are those based on L. Prandtl's semiempirical theory. Most frequently they are based on well-verified algorithms constructed for a plane boundary layer using semiempirical relations that contain the effects of curvature (Eq. (15), Sec. 2.6). The specific features associated with the change in the surface curvature radius (Eqs. (11) and (12)) as well as the effects of negative turbulent friction in the outer part of the boundary layer (Eq. (25)) are taken into account over the adaptation and relaxation lengths. In this case the results of calculation of turbulent flow and heat transfer are rather accurate [8, 11, 15-19] and reliable for practical application. Approximately the same approach to the calculation of curvilinear flows was developed in foreign works in which satisfactory results that take into consideration the effects of boundary layer adaptation and relaxation were obtained [12-14].

Recent investigations showed that in complex flows near curvilinear surfaces the principle of additivity of individual factors is satisfied with an accuracy sufficient for practical applications. This principle was developed by A. I. Leontiev and S. S. Kutateladze for plane flows. Using this approach, integral methods can be employed for calculating heat transfer and surface friction in gradient flows [7, 9, 10] in the case of injection into a boundary layer [17], as well as under conditions of nonisothermicity [8] and compressibility [15, 16]. Allowance for curvature is made in corresponding parameters of equations for the laws of friction and heat transfer, such as the critical parameters of flow separation and injection. This allows one to employ the algorithms used for calculations of plane flows (Secs. 2.4 and 2.5).

Methods based on complicated versions of the semiempirical theory of turbulence have been [6, 19] successfully employed for calculating curvilinear flows. In particular, allowance for the curvature effect in the generation term of the equation for ε (in the $k-\varepsilon$-model) gives a noticeable improvement in calculations of the heat transfer and hydrodynamics of semiinfinite jets [19].

In [26] an approach to calculation of heat transfer and friction near curvilinear surfaces has been developed that is based on the analogy to internal flow with flow twisting. In this approach the internal twisted flow in a tube corresponds to a flow past a "concave" wall, whereas the annular-channel flow corresponds to a flow past a "convex" wall (inner tube) and a "concave" wall (outer tube). In this approach, gradient flows correspond to a twisted flow in annular converging or diverging channels. In [26] it is shown that in this approach the geometric radius of the surface "curvature" of the helical line in a tube (of a "concave" surface) $R^* = R(1 + \tan^2\varphi)/\tan^2\varphi$ differs from the hydrodynamic radius $R^* = R\tan\varphi/(1 + \tan^2\varphi)^{0.5}$. Here R^* and R are the radii of the "concave" wall and the tube; $\tan\varphi = V_\varphi/V_x$; V_φ and V_x are the rotational and axial components of the twisted flow near the tube surface.

Use of the effective radius of a concave surface R^* makes it possible to perform calculations of friction and heat transfer using relations for an internal twisted flow in a tube [27, 28].

3. TOPICAL PROBLEMS

Analysis of the investigations carried out shows that problems of heat transfer and hydrodynamics under various conditions are considered in a great number of works. Nevertheless, many of the problems require further investigation. These are:

1) extension and deepening of modern concepts of the physical structure and laws of heat transfer in shear flows, semiinfinite jets, and gas screens under complicated boundary conditions (nonisothermicity, pressure gradient, injection and suction, external turbulence, etc.);

2) investigation of the laws governing adaptation and relaxation of the boundary layer under various boundary conditions;

3) investigation of the effect of two-phase flows (gas–solid particles, gas–liquid particles) on heat transfer and hydrodynamics;

4) investigation of the effect of significant curvature ($\delta/R_w > 0.1$) on local, turbulent, and integral characteristics of the boundary layer;

5) investigation of the laws governing transition from a laminar to a turbulent flow under complicated boundary conditions; further study of the physical structure of Görtler centrifugal instability;

6) development of methods for calculating heat transfer and hydrodynamics toward complicated semiempirical theories of turbulence, establishment of the relation with other types of flows in fields of centrifugal forces.

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NOTATION

c_f, coefficient of friction; Gö, Görtler number; $H = \delta^*/\delta^{**}$, form-parameter; $k = c_p/c_v$, specific-heat ratio; m, n, exponents for the temperature and velocity profiles; M, Mach number; Nu, Nusselt number; P, static pressure; Re, Reynolds number; R_w, surface curvature radius ($R_w > 0$, convex surface; $R_w < 0$, concave surface); Ri, Richardson number, s, width of the slit of the gas screen; T, temperature; u, v, w, velocity components; u_p, velocity of potential flow; u_{pw}, projection of the potential-flow velocity on the wall; x, y, z, rectangular coordinates; δ, thickness of the boundary layer; δ^*, δ^{**}, displacement and momentum loss thicknesses; δ_T^{**}, energy loss thickness; τ, friction; τ_t, turbulent friction; μ_t, turbulent viscosity; $\nu = \mu/\rho$, kinematic viscosity; $\theta = T - T_w/T_f - T_w$, dimensionless temperature; $\omega = u/u_\infty$; $\xi = y/\delta$; $\Lambda_0 = \delta(dP/dx)/t_{w0}$, parameter of the pressure gradient; η, efficiency of the gas screen; $\Psi_R = (c_f/c_{f0})_{Re^{**}}$; $\Psi_R^T = (St/St_0)_{Re_T^{**}}$. Subscripts: a, adaptation, m, maximum value; 0, entrance, flat plate; s, slit; f, flow; w, surface; 1, viscous sublayer; ∞, condition outside the boundary layer.

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